ARGENCON 2020. IEEE, RESISTENCIA, 2020.

Core Loss Prediction in Power Electronic Converters Based on Steinmetz Parameters.

Hernán Tacca.

Core Loss Prediction in Power Electronic Converters Based on Steinmetz Parameters

Hernán E. Tacca
Laboratorio de Control de Accionamientos, Tracción y Potencia (LABCATYP)
Departamento de Electrónica
Facultad de Ingeniería
UNIVERSIDAD DE BUENOS AIRES
htacca@fi.uba.ar

***Abstract*—Based on a modification of the Steinmetz equation, simple and general formulas suitable for magnetic power loss estimation in converter design are proposed. Using only the manufacturer data sheet parameters for sinusoidal waveforms, it is possible to estimate the core losses in symmetrical converters. A loss model is introduced to consider the loss increment caused by DC magnetization bias in asymmetrical converters, giving worst case criteria to design the magnetic components.**

Keywords—magnetic core losses, Steinmetz equation, power magnetic components, switching converters.

I. INTRODUCTION

In 1892 C. Steinmetz proposed an equation to calculate the energy lost in each hysteresis cycle [1]. According to his proposal, a similar expression was introduced to calculate all kinds of losses with sinusoidal waveforms, such as:

$$p_{V_{Stm}} = P_{Fe}/\mathcal{V}o\ell = k_S\ f^{\alpha}\ B_m{}^{\beta} \qquad (1)$$

which is usually referred as the Steinmetz equation [2], where: B_m is the maximum induction amplitude, f is the frequency of sinusoidal excitation and k_S , α , and β are constants found by curve fitting. Actually k_S , α , and β are not constant over the entire range of frequency and induction operation span. Therefore, they should be obtained from manufacturer curves using data close of the point of converter operation.

Sometimes, this involves a wide span of magnetic conditions. In such a cases, two alternatives may be adopted: Using one set of parameters suitable for the lower range of the frequency operation and other for the higher frequency range. The other approach may be to use some fitting formula, to adapt the parameters for considering the frequency dependence, as proposed in [3]:

$$k_S = A\ ln\ f_{[kHz]} + B \qquad (2.a)$$

$$\beta = \beta_O - C\ f_{[kHz]} \qquad (2.b)$$

where A , B , and C are constants to be determined using the core-material data sheet.

The Steinmetz equation and the data provided by magnetic material manufacturers is based only on sinusoidal excitation, whereas power electronic converters, new electric machines and motor drives have very different waveforms, which cause different kind of power losses [4], [5], [6].

Also, if DC magnetizing bias is present, an important additional core loss increment is found.

Therefore, a method for loss estimation in non sinusoidal operation is required.

Several prediction models are based in modifications of (1) to take account of arbitrary waveforms [7], [8], [9].

II. MODELS BASED ON STEINMETZ EQUATION MODIFICATION

A way to consider the effects of non sinusoidal waveforms is to include dB/dt in the expression of the instantaneous power losses, since these losses will be nul during the time intervals where B is constant.

Thus a loss model called *Natural Steinmetz Extension* [7] was proposed as:

$$p_{V_{NSE}} = \frac{P_{Fe}}{\mathcal{V}o\ell} = k_N (\Delta B/2)^{\beta-\alpha} \left\{\frac{1}{T}\int_0^T |dB/dt|^{\alpha}\ dt\right\} \qquad (3.a)$$

where:

$$k_N = k_S \Big/ \left[(2\pi)^{\alpha-1} \int_0^{2\pi} |cos\ \theta|^{\alpha}\ d\theta\right] \qquad (3.b)$$

Another loss model called *Modified Steinmetz Equation* [10] was proposed by defining an equivalent frequency f_{eq} :

$$f_{eq} = [2/(\pi\Delta B)^2] \int_0^T (dB/dt)^2\ dt \qquad (4.a)$$

Substituting (4.a) in (1) gives the *Modified Steinmetz Equation* as:

$$p_{V_{MSE}} = P_{Fe}/\mathcal{V}o\ell = k_S\ f_{eq}{}^{\alpha}\ f\ \left({}^{\Delta B}/_{2}\right)^{\beta} \qquad (4.b)$$

where $\Delta B = B_{max} - B_{min}$ is the peak to peak value of the induction.

A more accurate model valid for many different waveforms, named *General Steinmetz Equation* [9] was proposed as:

This work was financed by a grant corresponding to the UBACYT project 20020170100386BA from the University of Buenos Aires.

$$p_{V_{GSE}} = \frac{P_{Fe}}{V_{o\ell}} = k_i \left\{ \frac{1}{T} \int_0^T |dB/dt|^\alpha \left|B_{(t)}\right|^{\beta-\alpha} dt \right\} \quad (5.a)$$

where:

$$k_i = k_S \Big/ \left[(2\pi)^{\alpha-1} \int_0^{2\pi} |cos\theta|^\alpha \; |sin\theta|^{\beta-\alpha} \, d\theta \right] \quad (5.b)$$

An approximative formula valid when no minor loops are present is:

$$k_i = k_S / 2^\beta \, \pi^\alpha \{0.1758 + [1.08614/(\alpha + 1.354)]\} \quad (5.c)$$

In order to improve the consideration of waveforms with "minor loops" associated with the fundamental B-H loop, a software for taking account of its effects was developed [11] and the enhanced method was named *Improved General Steinmetz Equation (iGSE)*.

Further, in order to get equations expressing the core-losses as function of electrical variables of the converter circuit, a modified model called *Extended Steinmetz Equation* was proposed [12] (sacrifying some accuracy) as stated by:

$$p_{V_{ESE}} = \frac{P_{Fe}}{V_{o\ell}} = k_G \, f_{eq}{}^{\nu} \, (\Delta B/2)^\chi \, \left(\dot{B}_{rms}\right)^\zeta \quad (6.a)$$

where:

$$f_{eq} = \frac{1}{2} \, (1/\Delta B) \, \left(\frac{1}{T} \int_0^T \left| {dB}/{dt} \right| dt \right) = {\left|\dot{B}\right|_{av}} \Big/ {2 \, \Delta B} \quad (6.b)$$

$$\dot{B}_{rms} = \sqrt{\frac{1}{T} \int_0^T \left({dB}/{dt} \right)^2 dt} \quad (6.c)$$

$$\left|\dot{B}\right|_{av} = \frac{1}{T} \int_0^T \left| {dB}/{dt} \right| dt \quad (6.d)$$

For pure sinewaves: $f_{eq} = f$.

Substituting (6.b) in (6.a) one obtains:

$$p_{V_{ESE}} = k_m \, \left(\dot{B}_{rms}\right)^\gamma \, \left(\left|\dot{B}\right|_{av} \right)^\varepsilon \, (\Delta B/2)^\xi \quad (7)$$

For sinusoidal waveforms (7) should match the classical Steinmetz equation (1), yielding:

$$p_{V_{ESE}} = k_{ESE} \, \left(\dot{B}_{rms}\right)^{\alpha-\varepsilon} \left(\left|\dot{B}\right|_{av} \right)^\varepsilon (\Delta B/2)^{\beta-\alpha} \quad (8.a)$$

$$k_{ESE} = k_S / \left(\sqrt{2}\,\pi\right)^\alpha \left(\sqrt{8}/\pi\right)^\varepsilon \quad (8.b)$$

The equation 8.a will be defined as *Extended Steinmetz Equation (ESE)*. It includes a parameter ε which modifies the rise of the plotted function (loss vs. duty cycle). This parameter is determined looking for the best fit with experimental data. A good match is obtained adopting:

$$\varepsilon = 2 - \; 0.86 \, \alpha \quad (8.c)$$

Substituting (8.b) and (8.c) in (8.a) and dividing for the classical Steinmetz equation (1) it results:

$$M_{ESE} = \; p_{V_{ESE}} / p_{V_{Stm}} =$$

$$(1.234/4.863^\alpha) f^{-\alpha} \left(\left|\dot{B}\right|_{av} \right)^{2-0.86\alpha} \left(\dot{B}_{rms}\right)^{1.86\alpha-2} (\Delta B/2)^{-\alpha} \quad (9)$$

Considering that:

$$\dot{B}_{rms} \Big/ \left|\dot{B}\right|_{av} = V_{rms} / |V|_{av} = f_{f_V} \quad (10)$$

where f_{f_V} is the *shape factor* of the applied voltage, and:

$$V_{rms} = n \, S_{Fe} \dot{B}_{rms} \quad (11)$$

where n is the coil turns number and S_{Fe} is the core-section.

For unipolar waveforms:

$$\Delta B = (1/n \, S_{Fe}) \int_0^{T/2} v_{(t)} \; dt = \; |V|_{av} / 2 \, f \, n \, S_{Fe} \quad (12)$$

Therefore, substituting (12), (11), and (10) in (9):

$$M_{ESE} = 1.234 \; (0.8225)^\alpha \; \left(f_{f_V}\right)^{1.86\alpha-2} \quad (13)$$

Notice that to obtain M_{ESE} no assumptions were made neither on the type of converter nor in its waveforms, except for the unipolar waveform feature required. Also, to consider the minor loops influence a Simulink program was developed and it is available in [12].

III. Application to Symmetrical-Waves Converters

A. Rectangular Wave Converters (Fig. 1)

For a typical steel $\alpha \cong 1.3$ and for a square wave converter it is $f_{f_V} = 1$ which gives $M_{ESE} = 0.957$. Therefore, the core losses may be estimated using the manufacturer data measured for sinusoidal excitation, adopting $B_m \; = \; \Delta B/2$.

For a rectangular voltage waveform with the same peak factor than a sinusoid, it is:

$$f_{P_V} = V_m / V_{rms} \; = \sqrt{2} = 1/\sqrt{D} \quad (14)$$

Then, the duty cycle results $D = 0.5$ and f_{f_V} results:

$$f_{f_V} = V_{rms} / |V|_{av} = 1/\sqrt{D} \; = \sqrt{2} \quad (15)$$

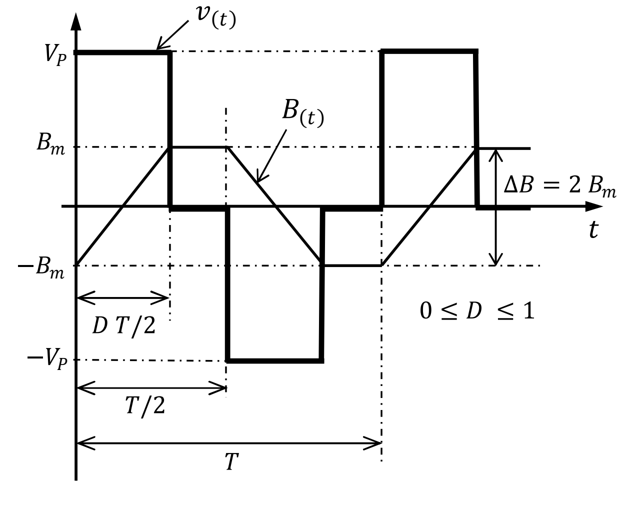

Fig. 1. Rectangular waveforms.

Therefore: $M_{ESE} = 1.106$.

If a material with $\alpha = 1.8$ were used, it should be:

$$M_{ESE} = 1.385 .$$

For most of laminated steels it is $\beta \cong 2$ so, to keep core losses constant, the induction B_m should be reduced regarding the value used for sinusoidal waveforms, as:

$$B_{m_{sq}} = 0.85\, B_{m_{sin}} \tag{16}$$

B. Unipolar PWM Sinewave Inverters

For an inverter using unipolar PWM sine wave synthesis (as shown in Fig. 2) the average transformer voltage must be:

$$V_P\, d_{(t)} = V_m \sin \omega t \tag{17}$$

where $d_{(t)}$ is the time dependent duty cycle required to produce the sinusoidal average value of the out waveform. So,

$$d_{(t)} = (V_m/V_P) \sin \omega t \tag{18}$$

If $V_m = V_P$, the rms voltage applied to the transformer becomes:

$$V_{rms} = \sqrt{\frac{2}{T}\int_0^{T/2} {V_P}^2 d_{(t)}\, dt} = V_P \sqrt{\frac{2}{T}\int_0^{T/2} \sin \omega t\, dt} =$$

$$= \sqrt{2/\pi}\; V_P \tag{19}$$

and the average rectified value results:

$$|V|_{av} = \frac{2}{T}\int_0^{T/2} V_P\, d_{(t)}\, dt = (2/\pi) V_P \tag{20}$$

Therefore the voltage shape factor is:

$$f_{f_V} = \sqrt{\pi/2} = 1.253 \tag{21}$$

From (13) this yields:

$$M_{ESE} = 0.786\, (1.2533)^{\alpha} \tag{22}$$

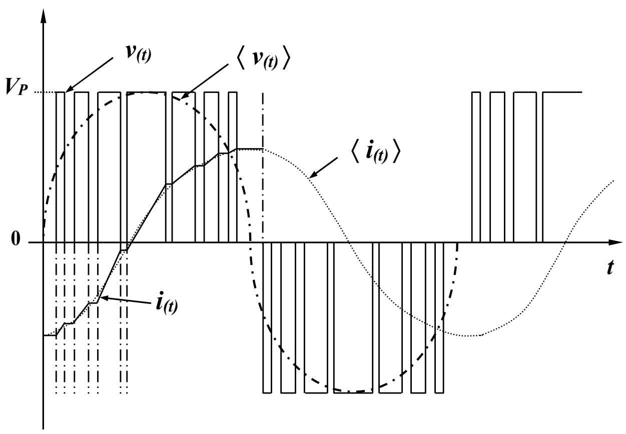

Fig. 2. Unipolar PWM waveforms.

For a typical lamination steel with $\alpha = 1.3$ this gives $M_{ESE} = 1.054 \cong 1$ and the data sheets curves for sinusoidal waveforms could be applied for power loss estimation.

C. Bipolar PWM Sinewave Inverters

In bipolar PWM, multiple B-H loops appear and the ESE will give results lower than the experimental ones.

To overcome this problem, the flux waveform is separated in individual loops without including inner loops, following the general procedure introduced in [11] and later adapted in [12], [21].

However, in this particular case, simplifications may be done to estimate the inner loops contribution to the core-losses increments.

In bipolar PWM there are multiple voltage commutations, each one corresponding to an inflexion point of the magnetizing current I_{M_i} (See Fig. 3) and so also to an inflection point of the induction B_i .

In Fig. 4 a switching cycle is detailed. There, the major B-H loop passes through points 2, 3, 5 and 6, while points 3, 4, and 5 form a minor loop.

Therefore, for this equivalent voltage waveform the shape factor will be the same obtained for unipolar PWM, so the major loop will have losses increased with respect to the case of sinusoidal driving according to a factor given by (22).

To compute the total losses, the minor loop losses have to be calculated in order to be added to the major loop ones. From Fig. 4 one obtains:

$$|dB/dt| = \Delta B_j/\Delta t_{n_j} = V_P/n_P\, S_{Fe} \tag{23}$$

$$\Delta t_{Z_{eq}} = 2\, \Delta t_{n_j} \tag{24}$$

As the derivative of the induction is a square wave, it follows that:

$$\dot{B}_{rms} = |\dot{B}|_{av} = |dB/dt| = \Delta B_j/\Delta t_{n_j} = V_P/n_P\, S_{Fe} \tag{25}$$

Substituting these values into the ESE expression and weighting the minor loop loss contribution by its time duration, yields:

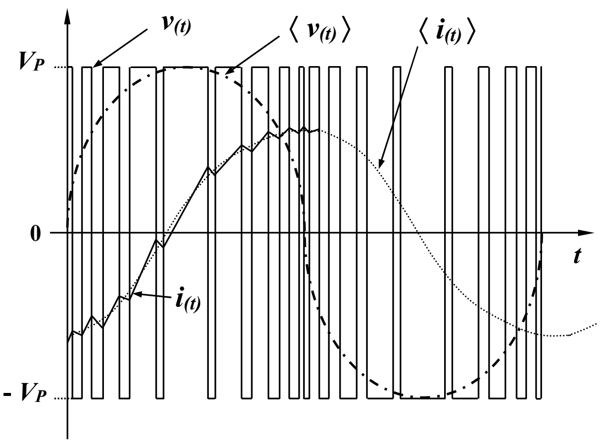

Fig. 3. Bipolar PWM waveforms.

$$p_{V_{ML_j}} = k_m \left(\Delta B_j/\Delta t_{n_j}\right)^{\alpha} \left(\Delta B_j/2\right)^{\beta-\alpha} \left(2\,\Delta t_{n_j}/T\right) \quad (26)$$

where,

$$k_m = 1.234\, k_S/(4.863)^{\alpha} \quad (27)$$

and $T = 1/f$ is the period of the sine wave to be synthesized.

Substituting (25) into (26) gives:

$$p_{V_{ML_j}} = 2^{\alpha-\beta+1}\, k_m\, f\, (V_P/n_P\, S_{Fe})^{\beta}\, \Delta t_{n_j} \quad (28)$$

From Fig. 4 it is:

$$\Delta t_{n_j} = (T_{SW}/2)\left(1 - \sin\omega\, t_j\right) \quad (29)$$

Substituting (29) in (28) gives:

$$p_{V_{ML_j}} = 2^{2(\alpha-\beta)} k_m \left[f/f_{SW}{}^{(\beta-\alpha+1)}\right](V_P/n_P\, S_{Fe})^{\beta}.$$

$$.\left(1 - \sin\omega\, t_j\right)^{\beta-\alpha+1} \quad (30)$$

Due to the symmetry of the induction waveform, the total minor loop losses will be twice the value of the rising part ones. Therefore:

$$p_{V_{ML}} = 2\, \sum_{j=1}^{n/2} p_{V_{ML_j}} = n\, \langle p_{V_{ML_j}} \rangle_{av} \quad (31)$$

where n is the number of minor loops per cycle of the synthesized sine wave:

$$n = f_{SW}/f \quad (32)$$

As $f_{SW} \gg f$ the discrete average may be approximated by the integral average, leading to the final simplified expression proposed in [12]:

$$p_{V_{ML}}/p_{V_{Stm}} =$$

$$= 0.47\,(0.8225)^{\alpha} \left(\frac{\pi}{2}\right)^{\beta} f^{\alpha-1}/(\beta-\alpha+1)^{0.75}\,(f_{SW}/f)^{\beta-\alpha} \quad (33)$$

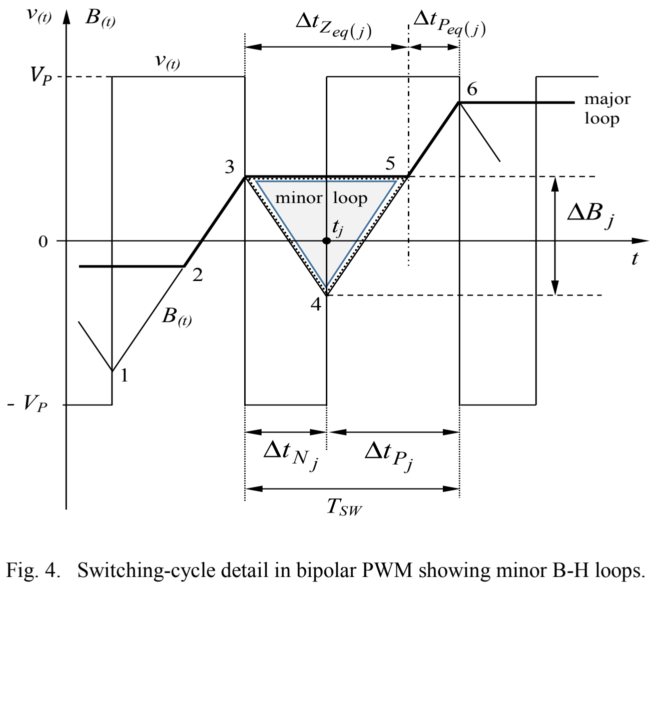

Fig. 4. Switching-cycle detail in bipolar PWM showing minor B-H loops.

Also, the maximum value of the minor loop amplitude normalized to the sinusoidal induction amplitude B_{S_m}, (amplitude of the sinusoidal component of the PWM modulated waveform) results:

$$\left|\Delta B_{ML_{max}}\right|/B_{S_m} = \pi\, f/f_{SW} \quad (34)$$

For $f = 50\ Hz$, with typical lamination values $\alpha \cong 1.3$, $\beta \cong 2$ and $f_{SW}/f = 200$, one obtains:

$$p_{V_{ML}}/p_{V_{Stm}} = 0.15$$

So, 15 % of additional core losses may be expected.

Taking account of the M_{ESE} calculated before for unipolar PWM, this leads to an overall multiplier of 1.21.

In order to keep the core losses equal to the ones expected for sinusoidal waveforms, the maximum induction should be reduced as:

$$B_{m_{PWM}} = B_{m_{sinus}}/(1.21)^{1/\beta} = 0.9\, B_{m_{sinus}} \quad (35)$$

This justify the common practice of adopting $0.9\,T \leq B_{m_{PWM}} \leq 1\,T$ for square waveform operation with steel laminations that could be used with $1.2\,T \leq B_{m_{sinus}} \leq 1.3\,T$ with sinusoidal waveforms.

On the other hand, for transformers made on ferrite, for typical values $\alpha = 1.3$, $\beta = 2.7$ and $f_{SW}/f = 200$, one obtains:

$$p_{V_{ML}}/p_{V_{Stm}} = 0.00124$$

and the minor loops influence may be neglected, designing the transformer as it would be done for unipolar PWM.

IV. Application to Asymmetrical-Waves Converters

Consider the half-bridge chopper shown in Fig. 5. There, R_O is the load that will take a DC current which introduces a magnetic bias in the core.

First, the unbiased operation will be analyzed (with R_O not connected).

In Fig. 6 the waveforms in the magnetic component are presented. There:

$$V_F\, DT = V_R\,(1-D)\,T \quad (36)$$

$$\left|\dot{B}\right|_{av} = 2\, f\, \Delta B \quad (37)$$

$$\dot{B}_{rms} = f\, \Delta B/\sqrt{D\,(1-D)} \quad (38)$$

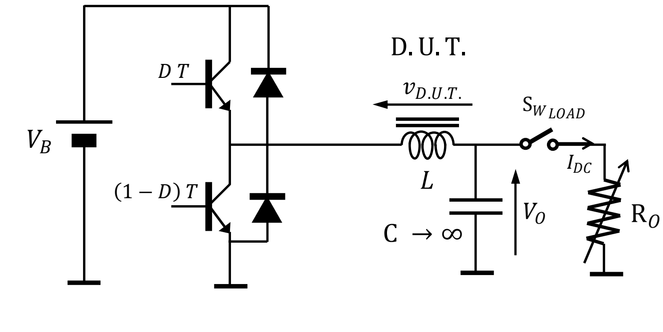

Fig. 5. Half-bridge chopper circuit.

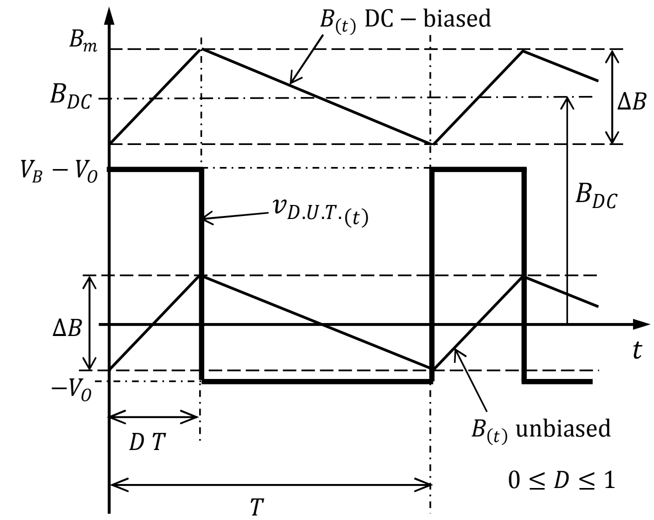

Fig. 6. Half-bridge chopper waveforms.

Substituting (37) and (38) in (9):

$$M_{ESE} = 4.936\,(0.2266)^{\alpha}/[D(1-D)]^{0.93\,\alpha-1} \quad (39)$$

which may be approximated as:

$$M_{ESE} = 4.9\,(0.2)^{\alpha}/[D(1-D)]^{\alpha-1} \quad (40)$$

From Fig. 6 and (36) the shape factor is:

$$f_{f_V} = V_{rms}/|V|_{av} = 1/2\,\sqrt{D\,(1-D)} \quad (41)$$

and substituting in (40) gives:

$$M_{ESE} = 1.225\,(0.8)^{\alpha}\, f_{f_V}{}^{2(\alpha-1)} \quad (42)$$

In Table I the values obtained from NSE, iGSE and ESE (using the simplified expression 40) are compared with experimental data from [7].

One may conclude that all the proposed modifications of the Steinmetz equation (NSE, iGSE, MSE and ESE) give results accurate enough for practical design purposes.

TABLE I *

MODIFIED STEINMETZ EQUATIONS: NORMALIZED POWER LOSS DENSITY RESULTS COMPARISON

MATERIAL	ΔB/2 [T]	f[kHz]	α	β	duty D	NORMALIZED POWER LOSS DENSITY			
						NSE	iGSE	ESE	measured
3F3	0.2	25	1.31	2.9	0.95	1.38	1.36	1.53	1.50
					0.90	1.20	1.18	1.26	1.18
					0.70	1.00	0.98	0.97	0.96
					0.50	0.96	0.95	0.91	0.88
	0.1	100	1.842	3.06	0.95	3.27	3.18	3.29	3.40
					0.90	1.95	1.89	1.92	1.77
					0.70	0.99	0.97	0.94	0.95
					0.50	0.86	0.84	0.81	0.81
N67	0.1	100	1.76	2.94	0.95	2.74	2.74	2.92	3.00
					0.90	1.74	1.74	1.80	1.76
					0.70	0.97	0.97	0.94	0.95
					0.50	0.86	0.86	0.83	0.78

* Experimental measures from [7]

V. CONSIDERING THE LOSS INCREASE CAUSED BY DC MAGNETIZING BIAS

Most of asymmetrical converters expose the magnetic core both to square waveforms and a DC magnetizing bias which increases the core-losses.

For silicon-iron Bozorth [13] gives diagrams plotting the losses as function of the alternating induction magnitude and the biasing induction.

For ferrites, experimental data may be found in [2], [14].

In some materials, when the AC part of the induction is lower than 100 mT, a small amount of DC bias slighty reduces the losses, but a further increase of the DC bias over 100 mT produces a remarkable increment of losses [15] - [18].

In [12] an empirical model based on experimental measures is proposed, giving a multiplier factor M_{DC} to multiply the modified Steinmetz equations in order to estimate the increment of losses due to the DC bias. This correction factor is:

$$M_{DC} = 1 + \kappa\,(|B_{DC}|/B_{Sat})^{\nu}\, e^{-\xi\left(\Delta B/2/B_{Sat}\right)} \quad (43)$$

where,

$\Delta B/2 = \hat{B}_{ac}$ is the equivalent amplitude of the induction to be introduced in ESE (or other modified Steinmetz equation),

B_{DC} is the DC bias induction,

κ, ν, and ξ are empirical parameters (almost constants) depending on the ferrite material.

Experimental measures done with materials N27, 3F3, 3C85 and 3C90 show that ν has not a critical value and good fitting is obtained adopting $\nu = 1.6$. Also, with some lack of accuracy, the parameter ξ may be expressed as:

$$\xi = (16/\kappa)^2 \quad (44)$$

This leads to:

$$M_{DC} = 1 + \kappa\,(|B_{DC}|/B_{Sat})^{1.6}\, e^{-(16/\kappa)^2\left(\Delta B/2/B_{Sat}\right)} \quad (45)$$

From (45) other simplified expressions were proposed in [12] in order to avoid the exponential function when inverse parameters determination were needed, or for linearization into a local operation area where the converter to be designed must work.

The parameter κ depends on the material, the temperature and the frequency, decreasing monotonally as the frequency increases (Fig. 7). Usually: $4 \le \kappa \le 9$.

In Fig. 8 the effect of the DC bias is shown, and the values obtained using (45) are compared with experimental measures.

Unfortunately, usually there are no parameters available in data sheets to consider the DC bias effect. If the actual value of κ is not available and measurements cannot be done, a worst case rough estimation of M_{DC} could be made adopting $\kappa = 9$, which gives:

$$M_{DC_{max}} = 1 + 9(|B_{DC}|/B_{Sat})^{1.6}\, e^{-3.16\left(\Delta B/2/B_{Sat}\right)} \quad (46)$$

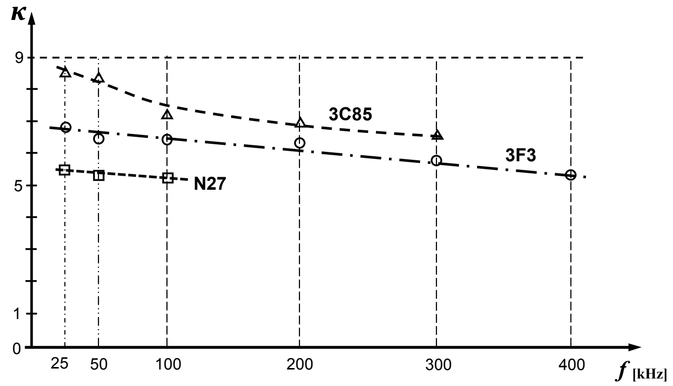

Fig. 7. Frequency dependence of parameter κ.

To prevent core saturation, the DC bias and the peak value of the AC part of the induction are related by:

$$(\Delta B/2) + |B_{DC}| \leq B_{Sat} \tag{47}$$

From (47) one may understand why is not usual to found simultaneously large peak to peak inductions (ΔB) and large DC components (B_{DC}). However, in critical continuous operation mode of asymmetrical converters it is:

$$B_{DC} = \Delta B/2 \tag{48}$$

VI. Application Examples

A. Application to a half-bridge chopper

Consider the circuit of Fig. 5 with: $f = 25\ kHz$,

$V_B = 50\ V, V_O = 5\ V, R_O = 1\ \Omega$, and $C = 330\ \mu F$.

The inductor is made on ferrite 3F3 with an inductance of 50 micro henrys which yields:

$\Delta I = 2\ A$, $I_{DC} = 5\ A$, and $D = 0.05$.

From the manufacturer data sheet one may consider:

$B_{Sat} = 0.35\ T$ and $\alpha = 1.8$.

From the above specified current values it results:

$$\Delta B/B_{DC} = 2/5 \tag{49}$$

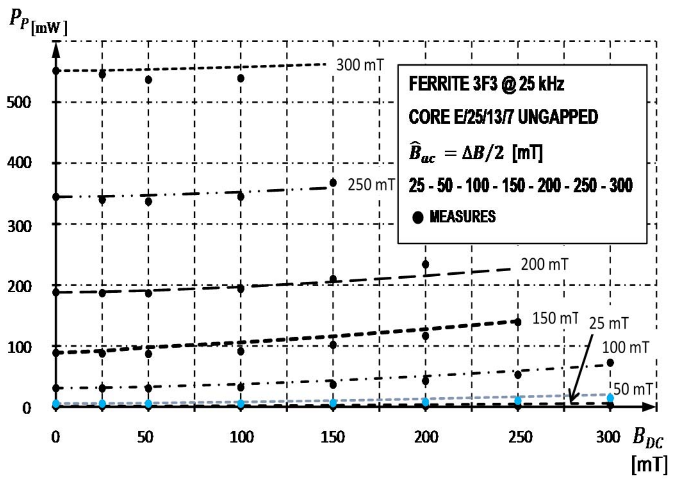

Fig. 8. Measured increment of losses caused by the magnetizing DC bias compared with values predicted using M_{DC}.

As design condition it will be adopted:

$$(\Delta B/2) + |B_{DC}| = 0.9\ B_{Sat} \tag{50}$$

Therefore, from (49) and (50):

$$\Delta B/B_{Sat} = 0.3 \tag{51}$$

$$B_{DC}/B_{Sat} = 0.75 \tag{52}$$

From (41) one obtains:

$$f_{f_V} = 2.294 \tag{53}$$

Then from (42):

$$M_{ESE} = 3.1 \tag{54}$$

To consider the DC bias effects, from measures done in [12], for the 3F3 material at 25 kHz it is $\kappa \cong 7$. Therefore, from (45) one obtains:

$$M_{DC} = 3.02 \tag{55}$$

Therefore the losses obtained from the manufacturer curves (measured for sinusoidal unbiased waveforms) using there $B_m = \Delta B/2$ should be multiplied by:

$$M = M_{ESE}\ .\ M_{DC} = 9.36 \tag{56}$$

If κ were unknown, using (46) a worst case estimation would be obtained as:

$$M_{DC_{max}} = 4.53 \tag{57}$$

Therefore:

$$M_{max} = M_{ESE}\ .\ M_{DC_{max}} = 14 \tag{58}$$

B. Application to a flyback converter operating in continuos mode

The circuit and the characteristic waveforms are shown in Fig. 9. In continuous mode the magnetomotive force (*m.m.f.*) is always greater than zero. The output voltage is:

$$V_O = (n_P/n_S)\left({}^{D}/_{1-D}\right) V_B \tag{59}$$

From Fig. 9.(*b*) one can obtain:

$$f_{f_V} = V_{P_{rms}}/|V_P|_{av} = 1/2\sqrt{D(1-D)} \tag{60}$$

Defining the transistor profit factor as:

$$f_{pr} = P_O/V_{CE_{max}} I_{C_{max}} \tag{61}$$

for a flyback converter operating in continuous mode one obtains:

$$f_{pr} = [1 - (\delta_i/2)]\ D(1-D) \tag{62}$$

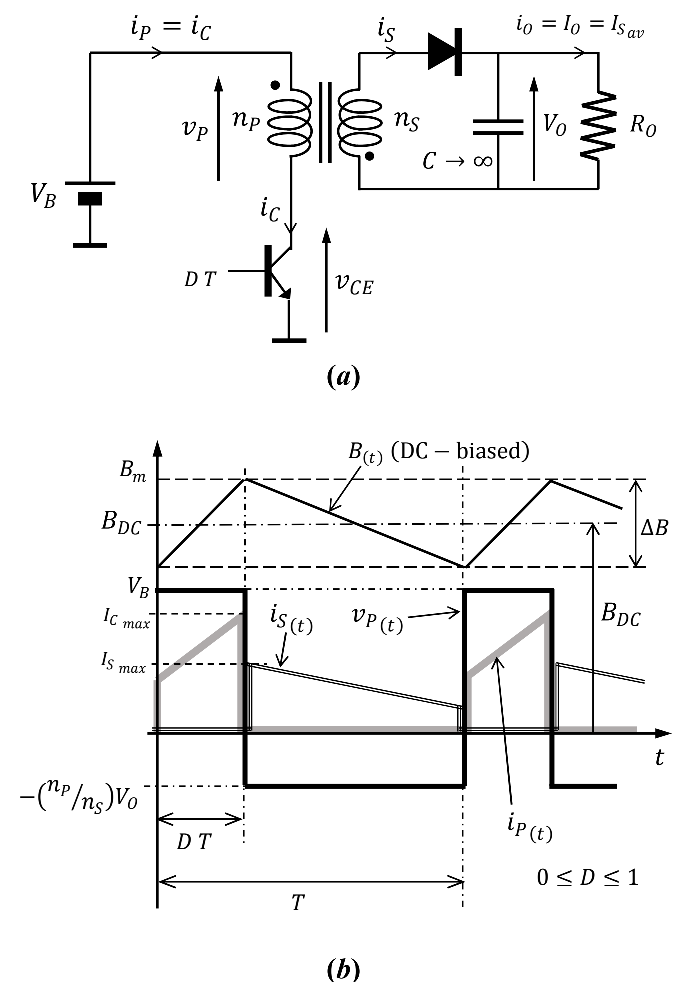

Fig. 9. Application to a flyback converter, (*a*) converter circuit, (*b*) waveforms in continuous mode operation.

where,

$$\delta_i = \Delta I_P / I_{P_{max}} \tag{63}$$

is a parameter used in converter design [19].

The optimum duty cycle is the one maximizing the profit factor of the transistor. That is:

$$D_{opt} = 1/2 \tag{64}$$

giving:

$$f_{pr_{opt}} = \frac{1}{4}\left[1 - (\delta_i/2)\right] \tag{65}$$

For the optimun duty cycle (60) gives:

$$f_{f_V} = 1 \tag{66}$$

and from (42) one obtains:

$$M_{ESE} = 1.225\,(0.8)^{\alpha} \tag{67}$$

On the other hand:

$$B_m = (\Delta B/2) + |B_{DC}| \tag{68}$$

From the Ampère law:

$$\Delta B / B_m = \Delta I_P / I_{P_{max}} = \delta_i \tag{69}$$

Therefore:

$$(\Delta B/2)/B_{Sat} = (B_m / B_{Sat})\left({}^{\delta_i}/_2\right) \tag{70}$$

From (68) and (70):

$$|B_{DC}| / B_{Sat} = (B_m / B_{Sat})\left[1 - \left({}^{\delta_i}/_2\right)\right] \tag{71}$$

Substituting (70) and (71) in (45) gives:

$$M_{DC} = 1 + \\ + \kappa\left[1 - \left({}^{\delta_i}/_2\right)\right]^{1.6} (B_m / B_{Sat})^{1.6}\, e^{-(16/\kappa)^2 \left({}^{\delta_i}/_2\right)\left({}^{B_m}/_{B_{Sat}}\right)} \tag{72}$$

For typical values,

$$f = 50\,kHz\,,\ \delta_i = 1/3\,,\ B_m / B_{Sat} = 0.8$$

adopting material 3C85 with $\alpha = 1.8$ and $\kappa = 8.5$, working at $D = D_{opt} = 0.5$ it results:

$$M_{ESE} = 0.82 \ \text{and}\ M_{DC} = 2.73$$

Then,

$$M = M_{ESE}\ M_{DC} = 2.24$$

which is the factor to multiply the losses obtained from the manufacturer curves for sinusoidal waveforms. For material 3C85 with:

$$f = 50\,kHz \quad \text{and} \quad B_{Sat} = 0.35\,T$$

one obtains,

$$B_{m\,sin} = \Delta B/2 = \left({}^{\delta_i}/_2\right)\left({}^{B_m}/_{B_{Sat}}\right) B_{Sat} = 0.047\,T$$

Therefore:

$$p_{V\,Fe} = M\, p_{V\,Fe_{sin}} = 22.4\,mW/cm^3$$

REMARK:

In many converter topologies it is:

$$f_{pr} = K_{(\delta_i)}\ D(1 - D) \tag{73}$$

Substituting (73) in (40) gives:

$$M_{ESE} = 4.9\,(0.2)^{\alpha}\left[K_{(\delta_i)}\right]^{\alpha-1} / {f_{pr}}^{\alpha-1} \tag{74}$$

Usually there is a trade off between δ_i and the transistor power factor. Decreasing δ_i allows to improve the transistor power factor but lead to an increase of the core volume [19].

Fixing both δ_i and the core volume, from (60) and (74) one may conclude that improving the profit factor adopting the optimum duty cycle will also reduce the core losses. By this reason, small duty cycles should be avoided and it is a common practice to adopt transformer isolated topologies even if output voltage isolation is not required.

VII. Conclusions

For transformers for PWM symmetrical converters, made with laminated steel, only a core-loss increment of 15 % should be expected regarding a classic transformer operating with sinusoidal voltage waveform.

For symmetrical high frequency switching converters the clasical Steinmetz equation, or the manufacturers curves measured using sinusoidal waveforms, may be used as a core-loss estimation accurate enough for practical first prototype design.

For those symmetric converters operating with asymmetric waveforms and superimposed DC magnetization bias [20], both loss multiplying factors proposed in this work should be used.

For asymmetrical converters, one of the modified Steinmetz equation must be applied when the duty cycle is far of the optimum value, and some prediction of the core-losses increment due to the magnetizing bias should be made when the DC bias is comparable to the magnitude of the alternating induction.

In most asymmetrical converter topologies, with duty cycle near 0.5, the experimental curves for sinusoidal operation will be accurate enough for loss estimation, but the DC bias influence must be considered.

Acknowledgment

Without the support and help of Professor Charles R. Sullivan from Thayer School of Engineering at Dartmouth College, this work would not have been possible. However, any errors or misinterpretations eventually found, are exclusively attributable to the author.

References

[1] C. P. Steinmetz, "On the law of hysteresis", AIEE Transactions, vol. 9, pp.3-64, 1892, Reprinted under the title "A Steinmetz contribution to the AC power evolution", introduction by J. E. Brittain, in Proceedings of the IEEE 72 (2) 1984, pp. 196-221.

[2] E. C. Snelling, “Soft ferrites: Properties and applications”, Butterworths, U.K.,1988.

[3] R. Ridley and A. Nace, “Modeling ferrite core losses”, Switching Power Magazine , January 2002.

[4] A. Van den Bossche and V. C. Valchev, "Inductors and transformers for power electronics", CRC Press - Taylor & Francis, U.S.A., 2005.

[5] W. G. Hurley and W. H. Wölfle, "Transformer and inductors for power electronics: Theory, design and applications", Wiley, 2013.

[6] M. K. Kazimierczuk, "High-frequency magnetic components", Wiley, 2009.

[7] A. Van den Bossche, V. C. Valchev and G. B. Georgiev, "Measurement and loss model of ferrites with non-sinusoidal waveforms," 2004 IEEE 35th Annual Power Electronics Specialists Conference (IEEE Cat. No.04CH37551), Aachen, Germany, 2004, pp. 4814-4818 Vol.6, doi: 10.1109/PESC.2004.1354851.

[8] J.Reinert, A. Brockmeyer, and R.W. A. A. De Doncker, “Calculation of Losses in Ferro- and Ferrimagnetic Materials Based on the Modified Steinmetz Equation”, IEEE Transactions on Industry Applications, vol. 37, no. 4, Jul./Aug. 2001.

[9] Jieli Li, T. Abdallah, and C. R. Sullivan, "Improved calculation of core loss with nonsinusoidal waveforms", in Conference Record of the 2001 IEEE Industry Applications Conference, 36th Annual Meeting, 2001, pp.2203-2210.

[10] M. Albach, T. Durbaum and A. Brockmeyer, "Calculating core losses in transformers for arbitrary magnetizing currents a comparison of different approaches," PESC Record. 27th Annual IEEE Power Electronics Specialists Conference, Baveno, Italy, 1996, pp. 1463-1468 vol.2, doi: 10.1109/PESC.1996.548774.

[11] K. Venkatachalam, C. R. Sullivan, T. Abdallah and H. Tacca, "Accurate prediction of ferrite core loss with nonsinusoidal waveforms using only Steinmetz parameters," 2002 IEEE Workshop on Computers in Power Electronics, 2002. Proceedings., Mayaguez, Puerto Rico, USA, 2002, pp. 36-41, doi: 10.1109/CIPE.2002.1196712.

[12] H. E. Tacca, “Extended Steinmetz Equation”, Thayer School of Engineering, Dartmouth College, Hanover, NH, Estados Unidos, Oct. 2002 (DOI: 10.13140/2.1.2837.5363).

[13] R. M. Bozorth, “Ferromagnetism”, IEEE Press (Classic reprints), 1994.

[14] A. Goldman, "Magnetic components for power electronics", Kluwer Academic Publ., U.S.A., 2002.

[15] F. Dong Tan, J. L. Vollin and S. M. Cuk, "A practical approach for magnetic core-loss characterization," in IEEE Transactions on Power Electronics, vol. 10, no. 2, pp. 124-130, March 1995, doi: 10.1109/63.372597.

[16] A. Brockmeyer, "Experimental evaluation of the influence of DC-premagnetization on the properties of power electronic ferrites," Proceedings of Applied Power Electronics Conference. APEC '96, San Jose, CA, USA, 1996, pp. 454-460 vol.1, doi: 10.1109/APEC.1996.500481.

[17] A. Brockmeyer and J. Paulus-Neues, "Frequency dependence of the ferrite-loss increase caused by premagnetization," Proceedings of APEC 97 - Applied Power Electronics Conference, Atlanta, GA, USA, 1997, pp. 375-380 vol.1, doi: 10.1109/APEC.1997.581478.

[18] V. C. Valchev, A. P. Van den Bossche and D. M. Van de Sype, "Ferrite losses of cores with square wave voltage and DC bias," 31st Annual Conference of IEEE Industrial Electronics Society, 2005. IECON 2005., Raleigh, NC, 2005, pp. 5 pp.-, doi: 10.1109/IECON.2005.1569013.

[19] H.E. Tacca, “Flyback vs. Forward Converter Topology Comparison Based upon Magnetic Design”, Eletrônica de Potência , Vol. 5, no. 1, May 2000, Brazil.

[20] S. A. Quinteros, H. E. Tacca, M. Pupareli, "2 kW DC/DC converter for battery charger with half-bridge conversion topology - Convertidor CC/CC de 2 kW para cargador de baterías, con estructura de conversión en medio puente", XXIII Congreso Argentino de Control Automático (AADECA 2012), Records in CD , 3-5 Oct. 2012, Buenos Aires, Argentina.

[21] Dartmouth Magnetic Component Research Web Site, http://engineering.dartmouth.edu/inductor.