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Mixture of fuzzy sets and examples of application in management

La mezcla de conjuntos difusos y sus aplicaciones en la administración

Le mélange d'ensembles flous et leurs applications dans l'administration

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RESUMEN

Como es bien sabido, las operaciones con conjuntos difusos se basan en cálculos usando los valores de funciones miembros o el grado de pertenencia de los elementos. En este trabajo, una nueva operación con conjuntos difusos se introduce y se denomina mezcla de conjuntos difusos, el cual opera no sólo con los valores antes mencionados, sino que también se basa en la mezcla de elementos de varios conjuntos difusos. A través de dos ejemplos desde la perspectiva de la gestión, una relacionada con el diagnóstico de habilidades de trabajo y otra relativa a la prospectiva de desarrollo local, se muestra la aplicación de la contribución teórica de la investigación, concluyendo con la demostración de su utilidad para la ciencia en la práctica.

Palabras clave: Mezcla de conjuntos difusos, conjuntos difusos mixtos, conjuntos difusos, matemática difusa.

RÉSUMÉ

C'est bien connu que les opérations des ensembles flous se basent sur des calculs en utilisant les valeurs de fonctions membres ou le degré d'appropriation des éléments. Dans cette étude, nous présenterons une nouvelle opération avec des ensembles flous nommée mélange d'ensembles flous dans laquelle on ne travaille pas seulement avec les valeurs mentionnées ci-dessus et qui si base sur le mélange d'éléments de différents ensembles flous. À travers 2 exemples dans une perspective de gestion, le premier associé au diagnostic d'aptitudes professionnelles et le deuxième à une perspective de développement local, nous montrerons l'application de la contribution théoriques de cette recherche, et finalement son utilisation pratique pour la science.

Mots-clés: Mélange d'ensembles fous, ensembles fous, mathématiques d'ensembles flous.

ABSTRACT

As is well known, operations with fuzzy sets are based on calculations using the values of membership or the degree belonging to the elements. In this paper, we will present a new operation with fuzzy sets, named mixture of fuzzy sets, which operates not only with the values mentioned above, but also with the mixture of elements of several fuzzy sets. Moreover, we will bring up two examples from the management perspective, in which one example is related to the job skills diagnosis, while the other to the local development prospective. We will also present the implementation of the theoretical contribution of our research and its utility, in practice, for science.

Keywords: Mixture of fuzzy sets, mixed fuzzy sets, fuzzy sets, fuzzy mathematics

JEL Classification: O10, O12, C00, C02, C65

Introduction

The historical need of the application of Mathematics to the social development, the economic activities, the environment and to sum up, to all areas of life, is something that no one with good judgement would be willing to contradict. Nowadays, science is related to all disciplines, and it needs the application of mathematic to prove its hypothesis correctly. We live times in which a scientist must be aware that he needs to apply Mathematics to particular sciences, as in ancient times, when every educated person knew that he must learn Latin.

Fuzzy mathematics are recognized as important by scientists due to its capability of adjusting to improbable or imprecise facts. It became quite explicit in the literature of the 1960s, when the second stage of the transition from the traditional to the modern view of uncertainty began. This stage is characterized by the emergence of several new theories of uncertainty, distinct from probability theory. These theories challenge the seemingly unique connection between uncertainty and probability theories, which had previously been taken for granted. They show that the probability theory is capable of representing only one of several distinct types of uncertainty.

It is generally agreed that an important point in the evolution of the modern concept of uncertainty was the publication of a seminal paper by Lotfi A. Zadeh (1965), even though some ideas presented in the paper were envisioned some 30 years earlier by the American philosopher Max Black (1937). In his paper, Zadeh introduced a theory whose objects—fuzzy sets—are sets with imprecise boundaries. The membership to a fuzzy set is not a matter of affirmation or denial, but rather a matter of degree.

The significance of Zadeh's paper was that it challenged not only probability theory as the sole agent for uncertainty, but the very foundations upon which probability theory is based: Aristotelian two-valued logic. When A is a fuzzy set and x is a relevant object, the proposition "x is a member of A" is not necessarily either true or false, as required by two-valued logic, but it may be true only to some degree, the degree to which x is actually a member of A. It is most common, but not required, to express degrees of membership in fuzzy sets as well as degrees of truth of the associated propositions by numbers in the closed unit interval (0,1). The extreme values in this interval, 0 and 1, represent, respectively, the total denial and affirmation of the membership in a given fuzzy set as well as the falsity and truth of the associated proposition.

Zadeh (2002) posed some simple examples of these forms in terms of a person's statements about when they shall return to a current place in time. The statement "I shall return soon" is vague, whereas the statement "I shall return in a few minutes" is fuzzy; the former is not known to be associated with any unit of time (seconds, hours, days), and the latter is associated with an uncertainty that is at least known to be on the order of minutes. The phrase, "I shall return within 2 minutes of 6 pm" involves an uncertainty which has a quantifiable imprecision; probability theory could address this form.

Several sources have shown and proven that fuzzy systems are universal approximations (Kosko, 1994; Ying et al., 1999). We should understand the distinction between using mathematical models to account for observed data, and using mathematical models to describe the underlying process by which the observed data is generated or produced by nature (Arciszewski et al., 2003). Fuzzy Mathematics, the branch of mathematics which many people recognize as the best way of dealing with imprecision, has proved that it is a thorough instrument to model reality that obviously lacks precision. However, theoretically speaking it is like any other science, susceptible to contributions.

Operations on fuzzy sets have no counterparts in classical set theory and, because of this, extensions of fuzzy sets into fuzzy logic allow for the latter to be much more expressive in natural categories revealed by empirical data or required by intuition (Belohlavek et al., 2002).

The operations with fuzzy sets outnumber the operations with classic sets. On the one hand, classic operations are performed as its union, intersection and complements undergo changes, this kind of operations are developed taking into account the membership degrees of the elements. On the other hand, particular operations for the fuzzy sets emerge, among the ones carried out on the same fuzzy set known as unary operations: normalization, expansion, concentration, contrast´s intensification and diffusion; all of them carried out also with regard to the membership degrees of the elements. Besides, with fuzzy sets it is possible to make operations of aggregation and comparison, executed with membership degrees too; for example, the distance between fuzzy sets gives a number as a result. Also, the principle of extension of Zadeh, operates with the grades of membership, but not with the elements of fuzzy sets.

Is also known that in life not all numbers and elements of fuzzy sets can be sometimes combined and be the source of new elements, but to date fuzzy mathematics does not provide operations executed from the elements, originating new ones.

For this reason, the objective of this work is in fact, starting from the discovery of operation among elements, to define it and to demonstrate that it is applicable to situations of the real world; also to show that such an application can be useful to decision making. This fact does not exclude that such operation should operate also with the membership degrees, due to what they mean for each element.

Mixture of fuzzy sets

The operation to be introduced generates a fuzzy set, whose elements are consequence of the combination of several sets´ elements, and their membership degrees are the result of the arithmetic average of the constituent elements´ membership degrees.

The referred operation is named mixture of fuzzy sets and it can operate of four different forms which are defined as follows.

Definitions

Given the fuzzy sets W1 = {(w11, f(w11)), (w12, f(w12),…, (w1i, f(w1i)}, W2 = {(w21, f(w21)), (w22, f(w22),…, (w2j, f(w2j)},..., Wm = {(wm1, f(wm1)), (wm2, f(wm2),…, (w2k, f(w2k)}; if it is obtained from these sets a set M = {(m1, f(m1)), (m2, f(m2),…, (mn, f(mn)}, such that each mi, i=1, 2,…,n, is a combination of elements that belongs to each W_k, k=1, 2,…,m, every element of each W_k is part of at least one mi, and the values of the f(mi) are the arithmetic average of the function values of the elements that form each mi, then the M set is called complete mixed fuzzy set and the operation that generated it is called complete mixture of W_k fuzzy sets. The M´s elements are called mixtures, the W_k´s elements are called fuzzy sets stores and to identify their elements it is used the appellative mixture's ingredients.

If from those same sets is obtained a set M = {(m1, f(m1)), (m2, f(m2),…, (mn, f(mn)}, such that each mi, i=1, 2,…,n, is a combination of elements that belongs to some (not all) W_k, k=1, 2,…,m, every element of each W_k is part of at least a mi, and the values of f(mi) are the arithmetic average (due to its sensitivity to extreme values) of the function values of the elements that make up each mi, then the set M is known as partially mixed fuzzy set type 1 and the operation that generated it is called partial mixture of the Wk fuzzy sets relative to the W_k stores.

If from those same sets is obtained a set M = {(m1, f(m1)), (m2, f(m2),…, (mn, f(mn)}, such that each mi, i=1, 2,…,n, is a combination of elements that belong to each W_k, k=1, 2,…,m, some (and not all) elements of each W_k are part of the mi, and the values of f(mi) are the arithmetic average of the function values of the elements that make up each mi, then the set M is known as partially mixed fuzzy set type 2 and the operation that generated it is called partial mixture of the W_k fuzzy sets relative to the ingredients that belong to the W_k stores.

If from those same sets is obtained a set M = {(m1, f(m1)), (m2, f(m2),…, (mn, f(mn)}, such that each mi, i=1, 2,…,n, is a combination of elements that belong to some (and not all) Wk, k=1, 2,…,m; some (and not all) elements of each W_k are part of the mi, and the values of the f(mi) are the arithmetic average of the function values of the elements that make up each mi, then the M set is known as totally incomplete mixed fuzzy set and the operation that generated it is called totally incomplete mixture of W_k fuzzy sets.

Example 1. Given the sets W1 = {(w11, 0.5), (w12, 0.85), (w13, 0.78)}, W2 = {(w21, 0.75), (w22, 0.9)} and W3 = {(w31, 0.85), (w32, 0.8), (w33, .88), (w34, 0.7)}.

From the following five mixtures there will be formed four mixed fuzzy sets.
m1 = (w11, w12, w22, w32), f (m1) = 0.7625
m2 = (w11, w21, w22, w31, w32), f (m2) = 0.76
m3 = (w13, w22, w33, w34), f (m3) = 0.815
m4 = (w13, w33, w34), f (m4) = 0.79
m5 = (w12, w22, w33), f (m4) = 0.88

The sets are
Ma = {(m1, 0.7625), (m2, 0.76), (m3, 0.815)}; which is a complete mixed fuzzy set, is the result of a fuzzy sets complete mixture from the stores W1, W2 and W3.

Mb = {(m1, 0.7625), (m2, 0.76), (m4, 0.79)}; which is a partially mixed fuzzy set type 1, is the result of a fuzzy sets partial mixture relative to the stores W1, W2 and W3.

Notice that the mixture m4 does not have any ingredient of W2 store.

Mc = {(m1, 0.7625), (m2, 0.76), (m5, 0.88)}; which is a partially mixed fuzzy set type 2, is the result of a fuzzy sets partial mixture relative to the ingredients that belong to the stores W1, W2 and W3.

Notice that even though the three mixtures have ingredients from the three stores, the W1 store´s w13 ingredient does not appear in any of the mixtures.

Md = {(m3, 0.815), (m4, 0.79), (m5, 0.88)}; which is an incomplete mixed fuzzy set, is the result of a totally incomplete mixture of the fuzzy sets W1, W2 and W3.

Notice that the mixture m4 does not have any ingredient of W2 store, and the W1 store´s w11 ingredient does not appear in any of the mixtures, the same happens with the W2 store´s w21 ingredient and the W2 store´s w31 and w32 ingredients.

It is obvious that it can be done a mixture from the ingredients of only one store, when this happens it is called an endogenous mixture.

An application to the work competence valuation

Before setting out the example, it is necessary to understand the selected definition of work competence, which expresses that: work competences are the identifiable and assessable set of aptitudes (knowledge, abilities, capabilities, skills, etc.) and attitudes (motivations, beliefs, values, behaviors, etc.) that allow the person to have a successful performance (Perez-Capdevila, 2012).

Example 2. It has been chosen an employee of an enterprise´s work area. It has been declared as necessary a set of aptitudes and attitudes that employees must have, and it has been done a valuation of those aptitudes and attitudes in that specific person.

So that, the person in question represents on the one hand an aptitude´ store and on the other hand an attitudes store.

In order to facilitate the reader´s comprehension, every store is expressed in form of a two columns chart, in one of which appear the ingredients and in the other the membership degrees.

TABLE 1. Store which ingredients are the aptitudes (W_{ap})

Aptitudes (Ingredients)	Membership degrees
Knowledge of the activity that he/she performs (AP1)	0.9
Ability to identify opportunities (AP2)	0.85
Capability to use time rationally (AP3)	0.75
Ability to make decisions (AP4)	0.85
Ability to prioritize (AP5)	0.8
Capability to control stress (AP6)	0.95
Capability to face up challenges (AP7)	0.9
Ability to handle conflicts (AP8)	0.95
Ability for communication (AP9)	0.95
Creativity (AP10)	0.85

TABLE 2. Store which ingredients are the attitudes (W_{ac})

Attitudes (Ingredients)	Membership degrees
He/she acts independently **(AC1)**	0.95
He/she keeps organized his/her workplace **(AC2)**	0.75
He/she measures and follows the course of his/her results **(AC3)**	0.95
He/she is concerned about inefficiency and waste of time **(AC4)**	0.85
He/she keeps calm in front of tense situations **(AC5)**	0.95
He/she contributes with ideas to have a positive impact on results **(AC6)**	0.9
He/she has good interpersonal relationships **(AC7)**	0.95
He/she expresses his/her opinions firmly **(AC8)**	0.95

Attitudes (Ingredients)	Membership degrees
Honesty **(AC9)**	0.95
He/she shows concern for achieving ambitious goals with quality **(AC10)**	0.95
He/she does not accept anybody to provoke his/her anger **(AC11)**	0.95
He/she promotes his/her organization´s values **(AC12)**	0.85
He/she respects his/her organization´s rules **(AC13)**	0.8
He/she respects the other´s opinions **(AC14)**	0.75
He/she shows self-confident **(AC15)**	0.95
He/she offers to challenging missions **(AC16)**	0.95
He/she is concerned for transmitting a good image of his/her organization **(AC17)**	0.8
He/she has feelings of mutual help **(AC18)**	0.9
He/she overcomes difficulties **(AC19)**	0.95

The elements of M mixture are precisely the competences, which are combinations of ingredients of one and other store, defined by a group of experts of the enterprise in question. Also, with the intention of facilitating readers´ comprehension, we will discuss separately each one of the following competences in charts: self-confidence (m1), self-control (m2), sense of membership (m3), initiative (m4), direct efforts to achievement (m5), planning and organization (m6) and team work (m7).

TABLE 3. Mixture m_1 with its membership degree to M

	W_{ap} Ingredients			W_{ac} Ingredients				Membership degrees
	AP1	AP7	AP8	AC1	AC8	AC15	AC16	
m_1	0.9	0.9	0.95	0.95	0.95	0.95	0.95	0.94

TABLE 4. Mixture m_2 with its membership degree to M

	W_{ap} Ingredient	W_{ac} Ingredients		Membership degrees
	AP6	AC5	AC11	
m_2	0.95	0.95	0.95	0.95

TABLE 5. Mixture m_3 with its membership degree to M

	W_{ap} Ingredient	W_{ac} Ingredients			Membership degrees
	AP1	AC12	AC13	AC17	
m_3	0.9	0.85	0.8	0.8	0.84

TABLE 6. Mixture m_4 with its membership degree to M

	W_{ap} Ingredients		W_{ac} Ingredients			Membership degrees
	AP1	AP2	AC1	AC6	AC19	
m_4	0.9	0.85	0.95	0.9	0.95	0.91

TABLE 7. Mixture m_5 with its membership degree to M

	W_{ap} Ingredients					W_{ac} Ingredients			Membership degrees
	AP1	AP2	AP3	AP4	AP10	AC3	AC4	AC10	
m_5	0.9	0.85	0.75	0.85	0.85	0.95	0.85	0.95	0.87

TABLE 8. Mixture m_6 with its membership degree to M

	W_{ap} Ingredients					W_{ac} Ingredients			Membership degrees
	AP1	AP2	AP3	AP4	AP5	AC2	AC3	AC4	
m_6	0.9	0.85	0.75	0.85	0.8	0.75	0.95	0.85	0.84

TABLE 9. Mixture m_7 with its membership degree to M

	W_{ap} Ingredients						W_{ac} Ingredients				Membership degrees
	AP1	AP5	AP6	AP7	AP8	AP9	AC7	AC9	AC14	AC18	
m_7	0.9	0.8	0.95	0.9	0.95	0.95	0.95	0.95	0.75	0.9	0.9

This way, the complete mixed fuzzy set is expressed as follows:

M = {(m_1, 0.94), (m_2, 0.95), (m_3, 0.84), (m_4, 0.91), (m_5, 0.87), (m_6, 0.84), (m_7, 0.9)}

According to this result it is assumed that the employee has got his self-control and self-confidence competences strengthened, but he is weak regarding sense of membership and planning competences.

As it has been noticed, this example determines work competences on the basis of an aptitudes and attitudes valuation, which allows to make decisions in order to strengthen the ones that are more weakened.

An application to products and services creation

Sometimes decision makers consider aims without making a preliminary analysis of the real possibility for achieving them. This example, unlike the previous one, is hypothetic; it just tries to illustrate the reasoning that can be followed to build the desired future, from the approximate knowledge of what you can get from what you have. It is not intended to get into disquisitions regarding the elements adjustment to the definitions about the used capitals, it is procured only to exemplify the mathematical procedure. In this particular example the mixture of fuzzy sets shows its prospective potential.

Example 3. Start from the supposition that in an X locality the highlighted elements of its human capital, its technological capital and its natural capital are known and it is necessary to know how far from these capitals products and services can be generated.

Proceeding analogously to the previous example, the three stores are set in a chart, and the membership degrees refer to the percent of what you have with respect to what is needed, expressed in the range (0, 1).

TABLE 10. Store which ingredients are the aptitudes (W_{ch})

Human Capital (Ingredients)	Membership degrees
Professors **(CH1)**	0.9
Doctors **(CH2)**	0.7
Agronomy Engineers **(CH3)**	0.8
Gastronomes **(CH4)**	0.95
Farmers **(CH5)**	0.9
Economists **(CH6)**	0.7
Masons **(CH7)**	0.75
Fishermen **(CH8)**	0.8
Scientifics **(CH9)**	0.45

TABLE 11. Store which ingredients are the aptitudes (W_{ct})

Technological Capital (Ingredients)	Membership degrees
Computers **(CT1)**	0.6
Teaching Aids **(CT2)**	0.8
Equipment for medical diagnosis **(CT3)**	0.6
Machinery for grinding rocks **(CT4)**	0.9
Technology for food production **(CT5)**	0.9
Internet **(CT6)**	0.5
Technology for fishing **(CT7)**	0.8
Agricultural machinery **(CT8)**	0.7

TABLE 12. Store which ingredients are the aptitudes (W_{cn})

Natural Capital (Ingredients)	Membership degrees
Land resource **(CN1)**	1
Medicinal plants **(CN2)**	0.8
Wild fruit trees **(CN3)**	0.3
Rivers **(CN4)**	0.4
Sea **(CN5)**	1
Sea fauna **(CN6)**	0.8
Quarry **(CN7)**	0.9
Nickel Mines **(CN8)**	0.9

In this case, the M mixture elements are products and services, which are ingredients combinations from the stores. The products and services about which it is needed to know to what extend they may be produced and offered respectively are described below:

m1 = Educational services
m2 = Health services
m3 = Gastronomic services
m4 = Fish production
m5 = Production of grain for construction
m6= Food production

Once more, with the intention of facilitating readers´ comprehension, it will be analyzed separately each one of the products and services in a chart.

TABLE 13. Mixture m_1 with its membership degree M

	W_{ch} Ingredients			W_{ct} Ingredients			Membership degrees
	CH1	CH6	CH9	CT1	CT2	CT6	
m_1	0.9	0.7	0.45	0.6	0.8	0.5	0.66

TABLE 14. Mixture m_2 with its membership degree M

	W_{ch} Ingredients			W_{ct} Ingredients		W_{cn} Ingredient	Membership degrees
	CH2	CH6	CH9	CT1	CT3	CN2	
m_2	0.7	0.7	0.45	0.6	0.6	0.8	0.64

TABLE 15. Mixture m_3 with its membership degree M

	W_{ch} Ingredients			W_{ct} Ingredient	W_{cn} Ingredients			Membership degrees
	CH4	CH6	CH8	CT5	CN2	CN3	CN6	
m_3	0.95	0.7	0.8	0.9	0.8	0.3	0.8	0.75

TABLE 16. Mixture m_4 with its membership degree M

	W_{ch} Ingredients			W_{ct} Ingredient	W_{cn} Ingredients		Membership degrees
	CH6	CH8	CH9	CT7	CN5	CN6	
m_4	0.7	0.8	0.45	0.8	1	0.8	0.76

TABLE 17. Mixture m_5 with its membership degree M

	W_{ch} Ingredients		W_{ct} Ingredient	W_{cn} Ingredient	Membership degrees
	CH6	CH7	CT4	CN7	
m_5	0.7	0.75	0.9	0.9	0.81

TABLE 18. Mixture m_6 with its membership degree M

	W_{ch} Ingredients				W_{ct} Ingredient	W_{cn} Ingredients		Membership degree
	CH3	CH5	CH6	CH9	CT8	CN1	CN4	
m_6	0.8	0.9	0.7	0.45	0.7	1	0.4	0.7

Notice that the M mixed fuzzy set generated is totally incomplete, because the mixture m1 did not use ingredients from the three stores and the ingredient CN8 was not used in any of the mixtures, being expressed as follows:

M = {(m1, 0.66), (m2, 0.64), (m3, 0.75), (m4, 0.76), (m5, 0.81), (m6, 0.7)}

From this point on it is deduced that grain production for construction is the one which has more perspective.

CONCLUSIONS

For the first time an operation between sets appear in Mathematics; it operates over the elements and transforms them. This definition in its four moments takes into consideration those elements with membership degree zero.

It is obvious that any of the four kinds of mixtures has utility in a decision making process in practice, and it facilitates interpretation of actual phenomena.

It is assumed from the second example that before building stores the model should be exhaustive enough to fit reality; it can be seen that, for instance, it cannot be conceived a medical service which lacks important ingredients (e.g paramedic staff, therapeutic resources, ambulances, etc.), which, given the case they do not exist, must be included in the store with zero membership degree, a step that would impact the arithmetic average decrease.

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