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A PROFESSIONAL DEVELOPMENT EXPERIENCE IN GEOMETRY FOR HIGH SCHOOL TEACHERS: INTRODUCING TEACHERS TO GEOMETRY WORKSPACES

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Abstract

This article deals with material designed within the framework of a teacher development project coordinated by the authors and aimed at providing support for in-service secondary school teachers who use or consider using Dynamic Geometry Software (DGS) to develop instructional sequences in mathematics classrooms. The project contains worksheets prepared as text support by authors and included in the Teacher's Guide (Fioriti et al., 2014a, 2014b, 2014c), which was provided to students in the context of a teacher development course. A premise of the course is that problem solving is a vehicle for students to learn mathematics meaningfully. To enable this kind of learning, the classroom should be organized as a learning community, and technology should be incorporated as a tool for expanding mathematical knowledge.

Keywords: Dynamic Geometry Software, Geometry, Secondary Level, Teacher Education, Teaching

Introduction

The importance for teachers to study the tasks that secondary school students will be asked to do, cannot be overemphasized. In this contribution, we present a work plan with a sequence of tasks to be carried out in the professional development of in-service, secondary mathematics teachers. The course introduces the teachers to a theoretical framework, includes tasks teachers should carry out during course sessions, and the didactic analysis of such tasks. The course provides opportunities for teachers to reflect on secondary school students' behavior while learning mathematics and when interacting with other students, teachers, and tasks. The course also provides opportunities for teachers to analyze the characteristics of tasks and activities to ensure that they can enable mathematical thinking by students.

With those aims in mind, we presented teachers with different problems relating to real-life and mathematical contexts. The underlying assumption was that the analysis and resolution of these problems in the teacher education classroom may usher the participants into a geometric working space (Kuzniak & Richard, 2014). This working space consists of interactions among*:*

1. A real space, as support material, with tangible and concrete objects;

2. A set of artifacts such as drawing instruments or construction software (GeoGebra, in this case);

3. A theoretical reference system based on definitions and properties (here, geometric space and area of 2D figures organized in such a way that teachers can ponder on how secondary students using technology to solve problems might be engaged in creating and validating their knowledge on geometry).

Theoretical Framework

Kuzniak and Richard (2014) point out that the teaching that favors the development of students' mathematical work at school requires a certain organization that the teacher is responsible to generate. Thus, in their professional education, it is important to provide teachers with the following:

- Opportunities for those who teach to be involved in formulating conceptual networks or mental schemes whereby teachers can ratify their beliefs and conceptions, and which can be used in class to allow students to produce their own schemes,
- Support materials with related content to encourage teachers to search for mathematical connections throughout the curriculum design,
- Teacher guides that allow teachers to write comments on the software they select and use in the classroom.

We designed a professional development course based on our own experience teaching with this framework (CEDE, 2015) and included work material in the Teacher´s Guide to be used as support for school texts in secondary school teaching (Fioriti et al., 2014a, 2014b, 2014c). The following principles were used to guide the design of the training course:

-The classroom is regarded as a community for the study of mathematics,

- Problems take place in mathematical contexts or occur in extra mathematical context as a learning engine,

- Conjectures and proofs are constitutive tasks of mathematical activity,

- Construction of models of a situation to be studied is the key in mathematics as it entails abstraction that reduces problems of complex nature to their essential characteristics. Students should identify a set of variables, relate them accordingly and transform those relations using any theoretical-mathematical system to produce new knowledge on the problems under analysis.

These guidelines form the framework of the teachers' professional development and how the sequence of activities and their management have been designed. Students decide how to solve the problems, search for the most relevant relationships between variables, and discuss the strategies used with other classmates. The teacher plays the role of a coordinator who chooses problems, encourages student-student as well as student-teacher interactions, and finally organizes students' ideas into a collective production. A teacher, as a real professional, believes that knowledge is produced as a result of the interaction between the problem and the student's peers (Fioriti, 2017).

The problems and activities proposed for didactic analysis are meant for teachers to debate how to manage the class in order to encourage students to try and produce different solutions, then discuss them, all the while dealing with the conceptual networks that involve the passage from arithmetic to algebra, the use of deductive reasoning as a way of justifying in geometry, and the use of different but equivalent representation systems as some of the activities that students beginning secondary school should do. At the same time, these problems and activities aim to encourage teachers to focus on ways of organizing class interaction and think about the validity, accuracy, clarity, and generalizations of students' mathematical statements.

The incorporation of computers into society has brought about such a cultural change that the way in which we see the world and live in it has changed. In the same way, the incorporation of computers in the classroom requires a cultural change in the way we study and acquire knowledge. This change affects mathematical knowledge in how it is studied as well as the organization and management of classroom instruction. Consequently, the teacher should have the skills to deal with this change (Bifano & Villella, 2012). The inclusion of technology in teaching is inevitable; it provides the opportunity to rethink activities and problems that make knowledge comprehensible, and it makes us aware of the powerful tools that we have at hand.

Given this scenario, the incorporation of technology in different ways (to do mathematics, to expand mathematical culture and, consequently, to expand knowledge) should be analyzed as part of the specialized training teachers acquire during their professional development.

A Management Model for Geometry Instruction

The proposal we have described includes topics of Geometry, which are characterized as the branch of mathematics that according to Villella (2008):

- can be *seen*. Geometric figures can be drawn or constructed using the properties that characterize them. This involves being aware of the difference between a diagram and a figure (Laborde,1998), a situation that requires a didactic examination (Charles-Pézard, Butlen, & Masselot, 2012),
- allows for *play*. The development of concepts at the core of the content networks to be studied through the manipulation of concrete objects gives learning an active, playful quality,
- *best connects to reality*. The 3-dimensional and 2-dimensional models it analyzes can be seen in material objects,

- *applies algebra concepts*. The same language and symbols in algebra are used to name and characterize geometric content,
- *helps to reason*. Its axiomatic structure develops thinking and helps generate the use of deductive reasoning in students (González & Herbst, 2006).

The proposed activities center around the connection between geometry and real life situations, which allows working with models and mathematical problems that require the use of geometric properties to justify the solution found. These activities enable teachers to think about the properties of geometric objects that are studied in secondary school. In this reflective process, a model (a mathematical representation for a non-mathematical object) is built, with theoretical developments whose properties become meaningful in terms of how they relate to the situation that originated them, and properties are studied and geometric objects are characterized according to reasoning and procedures of geometry itself.

The development of geometric concepts is presented in activities with the generic name *study* (Chevallard, 2009). We chose this way of identifying them as we believe the classroom will have the same qualities as a learning community when they are solved. This community is made up of a group of students coordinated by a teacher whose main task is to search for a solution to the problem given. In order to do this, the known data is used together with properties studied before or appearing for the first time, which makes the corpus of the answer discussed in groups. This classroom organization, as well as the use of the study content made in it, creates a particular environment that brings about different kinds of methods, qualities of the models used, and justifications of the steps followed as showed in this example:

A candy factory wants to design a pyramid-shaped wrapper for its products. An employee designed a paper like this one:

Answer:

a- How can you fold the paper in order to obtain a pyramid with a square as base?

b- Is your proposal the only possible one? Why?

Figure 1: An example of a problem

When content is set in this way, problem solvers need to apply the necessary conceptual networks to highlight the underlying geometric property in the problem and explain the resulting family of figures (Ferragina & Lupinacci, 2012). Therefore, the classroom becomes a place where debate, argumentation, and the use of properties to explain decision making are more relevant than using a figure as proof, which is common in secondary school classrooms. In addition, ideas about what steps students need to take and what elements they need to use flow freely. It generates communicative competence in the mathematics classroom since students have to justify elements chosen and steps taken (Villella & Ammann, 2012).

Technology as a Tool for Teaching Geometry: Incorporating DGS

The most basic concepts of geometry taught at school can be described as the combination of their properties with the use of relevant and irrelevant attributes (Vinner, 1982) that characterize them. In this identification or construction of a geometric concept, we can distinguish at least four elements:

1. The image of the concept: It refers to the concept as it appears in the mind of the subject who is studying it. It includes everything related to the concept that comes to mind, everything evoked when the word that names it is heard or when a picture or representation is seen.
2. The definition of the concept: It refers to the verbal form with which a certain notion is expressed (when it exists; it does not always include everything the learner knows about the geometric object in question). This definition is not necessarily mathematical.
3. A group of mental or physical operations, such as certain logical operations, that make a comparison with the mental picture easier.
4. Technology: in a broad sense, it refers to a socio cultural product that is useful as a physical and symbolic tool to relate to and understand the world around us.

The construction of the image of a geometric concept results from a mix of visual and analytical processes that are realized in two directions. On the one hand, there is the interpretation and comprehension of visual models. On the other hand, there is the ability to translate symbolic information into a visual image by using certain technology. The interpretation of the image is the product of visual processes where the irrelevant attributes of the visual component are obtained first and act as a distraction between our internal constructions and what is perceived by the senses (Villella, 2008). We believe that from its own conception, there is a certain technology in geometry that contributes to the definition of the geometric concept.

What aspects are to be considered when the translation to a visual image is made through DGS? In the same way that writing has restructured consciousness and the human mind has generated cognitive operations that had not been developed before it, new technologies transform subjectivity, capacity, and practices. (Evans & Levinson,2009; Rogoff & Lave, 1984; Smolensky & Legendre 2006)

Some teachers believe that with the incorporation of Information and Communication Technology (ICT) at school, there is a risk of limiting teaching. In this specific case, the risk exists if the teaching is limited to what can be seen on the screen: the geometric pictures, the graphic representations of functions, the result of calculations, and so on. In traditional mathematics instruction, where many teachers were and still are trained, it is common to focus on techniques, which usually appear before the problems that make them meaningful or needed. Mathematical software and calculators are tools that solve algorithms effortlessly and in the case of graphs and figures, DGS allows for some properties to be seen. Thus, it is necessary to modify classroom work and start solving problems that will enable students develop three cognitive processes of geometric activity:

1. Visualization, related to the representation of space and support material;

2. Construction, determined by the instruments used (GeoGebra) and geometric configurations;

3. Discursive, aimed at producing arguments and proofs (Kuzniak & Richard, 2014).

To overcome these processes, our teacher development course first provides meaningful concepts and then assigns work on the mathematical techniques.

Just as in oral language it was impossible to manage concepts associated with geometric figures, in written language it is impossible to think of dynamic geometry objects. In a teacher development classroom, this makes a good starting point for a discussion:

- A *technology* for dynamic geometry constitutes a new system of representation of geometric objects when using new ostensive objects: computerized pictures. These pictures differ from the ones made on paper precisely because of their dynamic nature. They can be moved and deformed on the screen while keeping the geometric properties that have been assigned by the construction procedure;
- *A production means* that uses a device (the computer) as a fundamental requirement for its use;
- *A particular language* that integrates not only the language of geometry but its articulation with computer language,
- *A semiotic tool* with particular characteristics that combines different models, particularly the geometry model in the software embedded in the computer language.

Using DGS allows for a new means of producing knowledge, with a specific language that must be known. Learning processes built in this way are encouraged through the design of teaching processes. Listed below are some of the goals students are expected to achieve:

1) *Interpret* the problem posed.
2) *Understand* the given information and establish relationships with the commands in the program.
3) *Formulate and test conjectures* about the concepts being taught.
4) *Design* strategies to confirm or refute conjectures.
5) *Summarize* information given.
6) *Communicate* the result of findings while trying to define what they managed to build.

Teachers' Professional Development

In this section, two activities are provided to exemplify what was described. The first one serves as an example of a model construction, and the second one exemplifies the study of the geometric object from the discipline itself. These activities will be used to describe the management of classroom work as well as the meaning that content is given through the use of technology.

We propose a collaborative task where it is important to consider what a teacher needs to know to develop a successful teaching process in which students gain more understanding about the nature of mathematical knowledge. With the analysis and resolution of this kind of teaching situation developed in the project, teachers are given the opportunity to discuss the different variables they should deal with in order to give students the possibility of reasoning, arguing, making conjectures, refuting, and modeling in order to provide meaning to the mathematical knowledge students are learning. In furtherance of this aim, we selected mathematical content in the specific context in which it would be used. Then, we analyzed the processes involved in teaching it, and made conjectures about how learning would be achieved. The whole procedure makes this mathematical content specialized and limited to teaching professionals. It is included in a sample about mathematics teachers' specialized knowledge MTSK (Muñoz-Catalán, 2015).

Applying mathematics to situations originating outside of mathematics

The following paragraph sets out a situation described as fiction from reality:

A farmer wants to install a water tank to provide water to the main house, the housekeepers´ house and a work shed. The tank should be as close to the main house as possible. However, due to the leafy trees surrounding the house which cannot be moved, the tank can only be installed 500 meters from the house. The idea is to place the tank at the same distance from the housekeepers´ house and the work shed. Where should the tank be erected?

In this example, the first decision to take leading to the solution of the problem above is to construct on the screen representations that model the two conditions set out in the problem:

a) The distance from the tank to the main house must be 500m,

b) The tank must be placed at the same distance from the housekeepers´ house and the work shed.

For that purpose, scales must be used and the points representing each element must be named (Figure2):

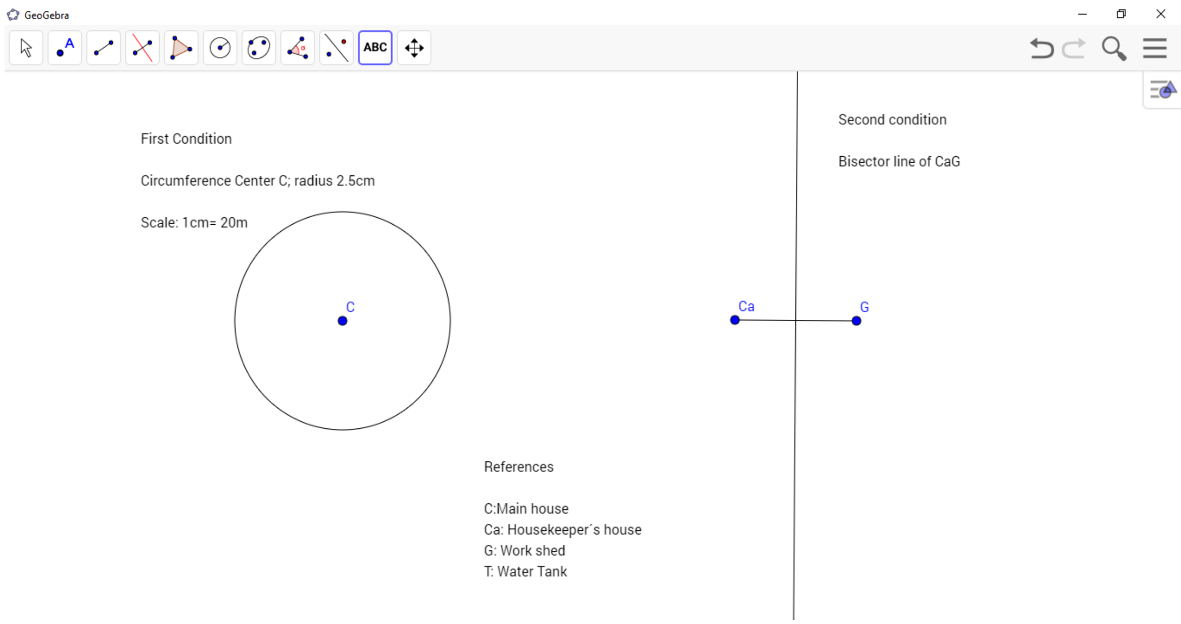


Figure 2: Possible answer attempt

Now, it is possible to establish the construction steps to be followed, what software tools are available, and what hidden conditions are being taken for granted by understanding the logic of the software.

Some teachers' (Tn) responses when they worked on this task:

T1: We need to draw a circumference. The problem says the tank must be placed at the same distance from the housekeepers´ house and the work shed. But, where do we draw the center of the circumference?

T2: Anywhere. The only important information is the radius' length.

T3: But we need to see them on the screen. So…point it near the center of the screen, please.

T1: Ok. Can you remember me the radius' length?

T2: I think it´s 500 m.

T1: So, We will need to use a scale.

T3: 1cm = 20m.Do you agree?

T1: Yes.

T2: Yes, it can be a good one.

T3: Use the command that shows circumferences to draw it…

T1: We need to use the second condition too!

T2: Uhh… You´re right. I´d forgotten it.

T3: Draw this figure near the other one. We´ll be able to compare the two figures all at once.

T1: It´s necessary but we need to use them at the same time in order to find the answer

T3: Uhm…let me see.…

All these activities need to be justified: the circumference has a given center and radius, the segment can have any length, however, the required line can only be its bisector, although the axes system cannot be visualized, the software assumes its orthogonal reference system, etc. The cognitive problem to be solved requires that both conditions be fulfilled simultaneously. The original screen (Figure 2) must be changed, so both conditions can lead to a model upon which conclusions can be drawn. The screen may show several pictures to be analyzed in terms of the dynamic nature of some points or figures. For example, if segment C_aG is moved, a possible figure of analysis is:

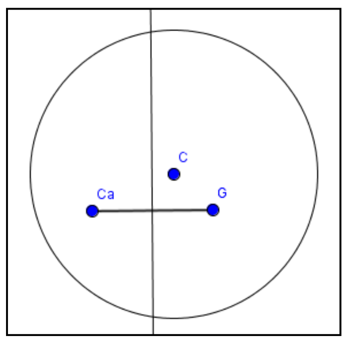


Figure 3: Dynamic study of the figure (case 1)

Some teachers' responses:

T1: This is a good answer (showing Figure 3).

T2: Um…it´s an answer, but not the answer!

T3: What do you mean?

T2: If I move G to the right, C_aG changes its length, then the bisectors line change too

T1: - Yes… and if we move C_a we obtain another line, so…

T3: Move them all around the screen, and let me see what happens...

T3: There are many answers…

T1: But…what happens if we choose a bisectors line by C?

T2. It´s a particular case.

T3: No, I think it´s the best answer, isn´t it?

Discussions, debates, and arguments based on certain properties arise by analyzing some possible answers to these questions: Does the result reflect the target model? What if the moving figure is another one and the screen obtained is the one below?

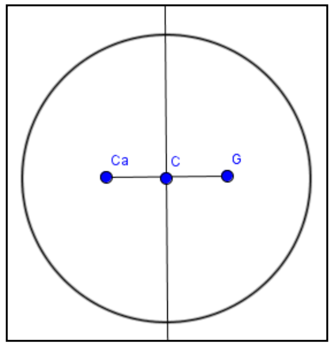


Figure 4: Dynamic study of the figure (case 2)

Answers may vary depending on the problem solver's perception. The dynamic nature of the point moving throughout the screen and the presence of other many infinite figures may change the answer. However, the logical reasoning leading to such an answer is still valid and so are the conclusions: The circumference of center C and radius 5cmrepresents the geometric locus of the points modeling the first condition of the problem, and the C_aG segment bisector is the geometric locus of the points modeling the second condition.

The figure of analysis becomes a knowledge object. This picture is no longer enough to solve the problem since the screen becomes the justification. Thus, the answer can only be found in the properties defining the geometric properties.

Some teachers 'responses:

T1: There are two conditions and two geometric properties: The circumference and the C_aG segment bisector.

T2: But, we need to use both of them to find the answer.

T3: If this is true, draw the only figure that uses both geometric properties.…

T1: Umm..Another problem. There are two points of intersection that satisfy both geometric properties.

T2: We need to study which of them is the appropriate one.

T3: I think both of them.

T1: Why?

T3: I can see it in the screen.

T2: No, it´s not enough.

T1: We need to justify… properties!, properties!…

The study of the teachers´ answers led to the construction of a model fulfilling both conditions. Such model being the one showing the intersection of both geometric loci requires another decision to be made: Which of the intersection points P_1 or P_2 will be considered point *T* (tank location)? Is it necessary to make this decision? Is it required by the formulation of the situation that gave rise to this study?

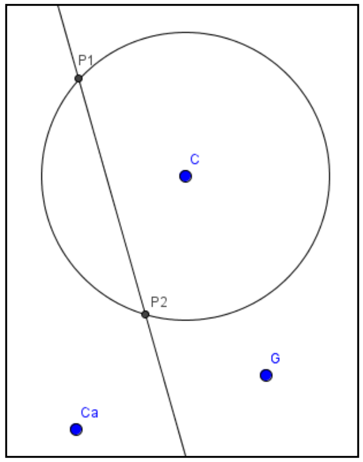


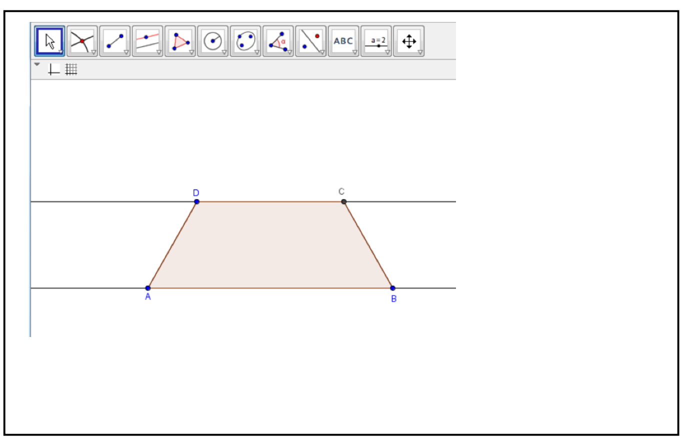
Figure 5: Dynamic study of the figure (case 3)

With these new questions, other questions arise which support the didactical analysis and make up the specific knowledge teachers must acquire as part of their training.

A situation within mathematics: Study of the geometric figure

In order to study a property of the isosceles trapezoid, we introduced the following activity:

In the GeoGebra screen below there is a trapezoid, which, by construction, is isosceles.



a) Determine the ratio between the areas of triangles *DAB* and *ACB*. Justify your answer.

b) If the trapezoid *ABCD* were not isosceles, would the answer to the question above still be valid? Explain why.

The first decision to make when solving the problem is to reproduce the figure on the screen, so both triangles can be seen (Figure6):

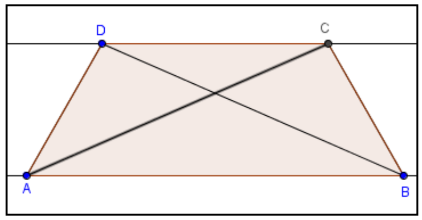


Figure 6: Reproduction of the trapezoid with the triangles drawn

Some teachers' responses:

T1: We need to reproduce the screen figure. This is an isosceles trapezoid, so we need the length of segment *DA* to be the same length of segment *CB*.

T2: Use circumferences!!

T1: Perhaps another tool is available. Let´s explore the tool bar.

T2: Yes...

T1: This is an isosceles trapezoid (showing Figure6 without the triangles).

T2: The problem says: "triangle *DAB* and *ACD*." Draw them, please.

T1: Here they are (showing Figure6).

T2: We need to determine the ratio between their areas. We need to calculate each one. So, base multiplies height and then we divide …

T1: Yes…but we don´t know the measurement and, if we move the baselines of the trapezium they will change. So, it´s not easy…

T2: Let me think…

The above dialogue leads to establishing the steps that must be followed in the construction and their pertinent justification: base lines are parallel; sides *DA* and *CB* have the same measurement. The cognitive problem lies in the lengths of *DA* and *CB* and in the area of *ABD* and *ABC*: They that are not measured directly and are visually considered equal. The solution entails designing a task that involves conceptual networks already studied: triangle height, bases, and similarities. In this case, both triangles have a common side (*AB*), and they both have the same height. The ratio between the areas is 1 as the areas are equal. Once question (b) is answered, the screen shows different pictures to be analyzed in terms of the dynamic nature of some of its vertices. For example, if vertex *A* is moved, possible figures of analysis (height is marked in dotted lines) are shown below (Figure 7):

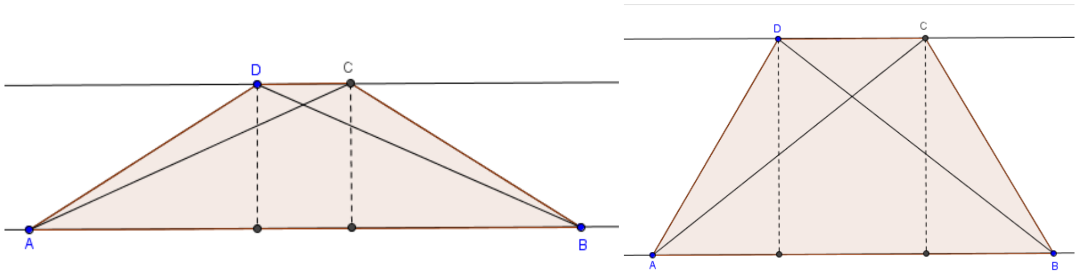


Figure 7: Dynamic modification of the figure

Some teachers' responses:

T1: The areas are equal. The ratio between them is one.

C (Coach): Why?

T2: We used properties!

C: Which ones?…

C: It´s OK. But, what happens if you move vertex *A*?

T2: Nothing!! The trapezoid is always isosceles.

C: Move it.

T2:I see it on the screen!

T1: Stop! You are changing the baseline length.

T2: But not the height.

C: So…

T2: Nothing happens.

C: The ratio between the areas doesn´t change, does it?

T1: Let me think, please.

C: OK.

T1: And if I move vertex *C*…

T2: It´ll be the same.

C: Try.

T1: It´s not the same.

T2: I agree…I need more properties!!

The questions arising from the analysis of these figures are: Does the ratio between the areas change because the shape also changes? If the moving point is a different one, could another figure be obtained? Once more, the presence of many other infinite figures may change the answer. However, the logical reasoning leading to such an answer is still valid and so is the conclusion: The triangles have the same area. The figure to be analyzed is not enough to solve the problem since the screen becomes the justification; thus the answer can only be found in the properties defining the area and the triangle similarities.

If we compare the areas of triangles *AOD* and *COB*, with *O* the point of intersection of the diagonals, can we reach the same conclusion? Upon exploring the figures obtained when *O* is located in different places on the screen, the shape of the figure changes but the ratio of the areas is the same. *AOB* is part of the two triangles compared in the original problem, by subtracting it from the new triangles to be compared, "the same area" is subtracted; thus such areas are equal. Now, we may wonder: What properties are brought into play if we compare triangles with similar areas but with different bases and height?

When considering the problem, teachers may raise doubts about the mathematical knowledge they think they possess after analyzing the ratio between areas not measured directly and studying the ratio reaction after obtaining different figures of analysis. Furthermore, the need to use properties that go beyond what is seen on the screen challenges the knowledge teachers have on geometric structure, and sets in motion a more active way of solving mathematical problems.

An analysis of specialized content knowledge for teaching (Muñoz-Catalán, 2015) prompts the assumptions teachers make about the way geometry is taught and learned. In our case, we add the use of DGS, which besides adding dynamism to answers leads to a series of assumptions regarding the geometric object of study that need to be confirmed. The problem described above is meant to analyze the specific knowledge about geometry each teacher has, considering what each teacher knows about Geometry, and the specialized content knowledge for teaching (SCK; Ball & Bass, 2009) each teacher has acquired. This allows teachers to interrelate content, to weigh student reasoning and mathematical solutions, and to recognize the validity of the arguments that may arise.

Conclusions

In our proposal, the mathematical content to be learned includes problems from two different work contexts: the modeling of a real situation that requires the construction, study and analysis of a model so that the conclusions drawn may be applied to solving the situation from which it originated, and the study of figures within mathematics where the use of properties and the construction using valid reasoning lead to the targeted solution.

The presentation of these two different types of problems in the teacher training classroom is relevant for teachers as it allows them to study the underlying structure of geometric working spaces: An epistemological level, linked to mathematical content, and a cognitive level, linked to visualization, construction, and proof. In order to articulate these two levels and obtain sound mathematical work, we propose discussing with teachers the development of figural genesis, relating space and figures (epistemological level) with visualization (cognitive level), instrumental genesis, relating artifacts (DGS, paper and pencil, etc.) from the epistemological level with construction (cognitive level), and discursive genesis, relating the reference framework (epistemological level) with proof (cognitive level; see Kuzniak & Richard,2014).

For teachers, this articulation into two levels includes a wide range of teaching situations that lead to the development of a mathematical work space inside the classroom and the use of a learning community. Our interest in the use of DGS lies in its capacity to support the discussion with teachers about the acquisition and construction of geometric knowledge in the secondary school classroom.

In addition to the specific knowledge of teachers, the work proposed supports reflection about the mathematical performance of secondary school students at the moment of studying and how they solve specific geometric situations. Some of the points discussed with teachers include how students design models, use metaphors to communicate findings, and organize explanations and reports to communicate discoveries and verifications. Other times, the points discussed were how students design strategies to find solutions justifying the procedure used, select material, spend time, appreciate both their own and their classmates' performance, accept mistakes, and correct the models used. It is important to analyze how students transfer the knowledge acquired to other learning contexts analyzing the wrong ideas acquired from the physical representation of objects, realize the double status of geometric objects, since the drawing of an object is sometimes considered the object itself, and the need of a description characterizing the object with the purpose of removing any ambiguity related to its representation.

The management of instruction—the design, performance, assessment, and generalization of teaching strategies performed by the teacher—leads to a process of negotiating the interests of students and teachers, where teachers act as stewards of a learning environment. The interests of students are based on meaningful content to be developed and on the naturalization of the use of DGS in the world of mathematics instruction. The interests of the teachers are based on the epistemology of the given content. In this negotiation, teachers act as natural mediators between the content and students; while teachers design and pose problems to be solved, students develop strategies to solve such problems where both teachers and students are part of a classroom project.

Regarding mathematics in secondary school, the use of DGS generates several ways to introduce tests as an unavoidable element of conceptual networks that are essential to the learning process. Teachers can suggest situations for graphic and dynamic research for students to analyze the behavior of geometric objects and the relations among them and thus, understand mathematical concepts and procedures, to justify and to do some more formal tests. DGS helps teachers lead a learning process by dealing with contradictions and causing students to learn about the formal demonstration process, explain why a result is mathematically true, communicate mathematical relations and properties used and discover by manipulating dynamic objects develop logical and abstract thinking, systematize by organizing results into a deductive system of axioms and theorems and to discover and construct mathematical knowledge.

In our proposal, the technological tool is used as a means to explore different types of graphic representations interactively. Thus, geometric objects can be constructed out of a variety of primitive objects (points, segments, lines, etc.) in this creative environment thought by the mathematics teacher as a professional.

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